Nuclear equation of state and finite nucleon volumes

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Abstract. It is shown how the Equation of State (EoS) depends on nucleon properties inside Nuclear Matter (NM). We propose to benefit from the concept of enthalpy in order to include volume corrections to the nucleon rest energy, which are proportional to pressure and absent in a standard Relativistic Mean Field (RMF) with point-like nucleons. As a result, the nucleon mass can decrease inside NM, making the model nonlinear and the EoS softer. The course of the EoS in our RMF model agrees with a semi-empirical estimate and is close to the results obtained from extensive DBHF calculations with a Bonn A potential, which produce an EoS stiff enough to describe neutron star properties (mass–radius constraint), especially the masses of "PSR J16142230" and "PSR J0348+0432", known as the most massive ($\sim 2M_{\odot}$) neutron stars. The presented model has proper saturation properties, including a good value of compressibility.

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Taking into account the thermodynamic effects of pressure in finite volumes, we will attempt to describe how the energy per nucleon $\varepsilon = M_A/A$ and the pressure evolve with NM density ϱ in a RMF approach [1, 2, 3, 4, 5]. The original Walecka version [1] of the linear RMF introduces two potentials: a negative scalar g_SU_S and a positive vector $U_V = g_V(U_V^0, \boldsymbol{\theta})$ fitted to the nuclear energy ε at the equilibrium density $\rho = \rho_0$. The EoS for this linear, scalar-vector (σ, ω) RMF model [1, 2] matches a saturation point with too large compressibility $K_A^{-1} = 9\varrho^2 \frac{d^2}{d\rho^2} \varepsilon \sim 540 MeV$ and is very stiff for higher densities where the repulsive vector potential starts to predominate over the attractive scalar part. Nevertheless RMF models produce, after the Foldy-Wouthuysen reduction, a good value of a large spin-orbit strength [1] at the saturation density which depends on $(g_V U_V^0 - g_S U_S)$. The dynamics of potentials in the RMF approach are discussed e.g. in [7] in four specific mean-field models [1, 2, 3, 4]. In the ZM model [4] a fermion wave function is re-scaled and interprets a new, density dependent nucleon mass. It starts to decrease from $\varrho = 0$ and at the saturation point $\varrho = \varrho_0$ reaches 85% of the nucleon mass M_N . But the nucleon mass replaced at the saturation point by a smaller value would change the deeply inelastic Parton Distribution Function [6], shifting the Björken $x \propto (1/M_N)$. Such a shift means that nucleons will carry 15% less of the longitudinal momentum, which should be compensated by an enhanced contribution from the meson cloud for small x < 0.3 to describe the EMC effect [7, 8]. There is no evidence for such a huge enhancement [9] in the EMC effect for small x. Moreover, the nuclear Drell-Yan experiments [10, 7], which measure the sea quark enhancement, have been described [11] with a small 1% admixture of nuclear pions and the M_N unchanged. Thus the deep inelastic phenomenology indicates that the change in the nucleon mass at the saturation density is rather negligible. Another nonlinear extension of the RMF model [3, 12] assumes self-interaction of the σ -field with the help of two additional parameters fitted to $K_A^{-1} \approx 250 MeV$ and an effective mass $M_N^* = M_N + g_S U_S$. These modifications of the scalar potential give a softening of the EOS but are unstable for high densities ([13]) and another nonlinear version has been proposed ([14]). Modern RMF calculations [15, 16, 17, 18, 19] include the important Fock term of the nuclear mean field [16] and have adjusted the EOS, fitting more meson fields (ρ for an isospin dependence) including the octet of baryons. The introduction of couplings of the meson fields to derivatives of the nucleon field is a purely phenomenological approach [18].

There are existing works [20, 21, 22] on excluded volume "effects" in which nucleons with eigenvolume Ω_N are treated as point-like particles in the available volume $\Omega_A - A\Omega_N$; therefore vector and scalar densities are scaled respectively. The properties of the ground state obtained[20] are more realistic due to the additional repulsion from excluded volumes, resulting in the reduction of the vector and scalar fields. In a similar approach Costa et al. discuss[21] excluded volume effects on nuclear matter properties and neutron star maximum masses and radii for the different RMF models. They found that only nonlinear parameterizations retain the proper NM compressibility. Using a different model Rocha et al.[22] introduce an attractive phenomenological Nucleon-Nucleon (NN) potential in order to get saturation properties for the repulsion

generated by excluded nucleon volumes. However, a non-relativistic approach to the NN interaction does not allow the use of this model for dense nuclear matter inside neutron stars. There are also nuclear bag models[23, 24, 25] which discuss the density dependent bag constant and radius in the nuclear equation of state. Kagiyama et al.[23] have obtained a nucleon bag heavier and much smaller for higher densities; such a possibility will be discussed here as rather unrealistic. The successful Quark-Meson-Coupling RMF models[24, 25] will be elaborated on and compared in section 1.2.

Our objective is to examine the main approximations of the RMF, namely the constant nucleon mass and the constant nucleon volume in the compressed nuclear medium. The nuclear EoS, in particular compressibility, depends on the NN interaction but also on the compressibility K_N^{-1} of quark matter confined inside nucleons. The novel feature of our work is a direct volume contribution of pressure to the rest energy of a nucleon, missed in previous works[20, 21, 22] on excluded volume effects with the constant nucleon radius and also absent in Quark-Meson Coupling (QMC) models [24, 25]).

Any extended object inside a compressed medium (like a submerged submarine) needs extra energy to preserve its volume. Thus from a "deep" point of view, a finite pressure correction should be taken into account in the RMF calculations. To describe the possible dependence of the nucleon mass on the nucleon volume in a compressed medium we adopt a nucleon bag model. For fixed pressure and zero temperature it is easy to show (see the first paragraph in the next section), that definitions of a chemical potential μ or a Fermi energy, have the same energy balance as an average single particle enthalpy. But enthalpy contains explicitly an interesting term, the work of the nuclear pressure p_H in the nuclear/nucleon volume, which will be investigated. We consider NM for zero temperature, therefore our choice of enthalpy corresponds to a Gibbs free energy with independent pressure p_H in favor of a Helmholtz free one (here an internal energy M_N) with the volume as an independent variable. Our results are independent [26] of this choice; like the expression for the chemical potential μ in (3). The simplest (σ, ω) model [1, 2], which is too stiff with point-like nucleons, will be used to obtain clear conclusions. We will neglect nuclear pion contributions above the saturation point. Dirac-Brueckner calculations show that the pion effective cross section, in the reaction of two nucleons $N + N = N + N + \pi$, is strongly reduced at higher nuclear densities above the threshold [27] (also with RPA insertions to the self energy of N and Δ [28]). We restrict our degrees of freedom to interacting nucleons. Further work for finite temperature is planned.

1. Nuclear Enthalpy

At the beginning, let us consider effects generated by a volume of compressed NM starting with A nucleons which occupy a volume $\Omega_A = A/\varrho$. They have to perform the work $W_A = p_H \Omega_A$ necessary to keep the space Ω_A inside the compressed NM against the

nuclear pressure p_H . The nuclear enthalpy at zero temperature T is given by:

$$H_A \doteq M_A + W_A = M_A + A \frac{p_H}{\rho} \tag{1}$$

$$p_H \doteq -(\partial M_A/\partial \Omega_A) \mid_{T=0}$$
 (2)

and contains, besides the nuclear mass as an internal energy, the necessary work. Taking appropriate thermodynamical derivatives with respect to A, we get the following relations between the chemical potential μ and the enthalpy for $A \to \infty$:

$$\mu \doteq (\partial M_A/\partial A)_{\Omega_A} \equiv (\partial H_A/\partial A)_{p_H} = \varepsilon + \frac{p_H}{\rho} = H_A/A. \tag{3}$$

The same relation fulfils a nucleon Fermi energy with a Fermi momentum P_F

$$E_F \doteq P_N^0(P_F) = (\partial M_A/\partial A)_{\Omega_A} = \varepsilon + p_H/\varrho = \mu, \tag{4}$$

well-known as the Hugenholtz-van Hove (HvH) relation[26], also proven in the self-consistent RMF approach [29].

1.1. Finite nucleon volumes in NM

There are two different media inside nuclear matter. The hadron medium - where the NN interaction operates inside $\Omega_{A-} = \Omega_A - A\Omega_N$ does not include nucleon volumes Ω_N filled with a confined quark matter with density $\varrho_N = N_q/\Omega_N$. Previous work[20, 21] on the effect of excluding nucleon volumes assumed constant nucleon volumes while the present work also discusses a decreasing volume Ω_N for a finite nucleon compressibility $K_N^{-1} > K_A^{-1}$. Let us justify the possible modification of K_A^{-1} for the saturation density. Pressure p_H and nuclear compressibility K_A^{-1} are defined in the macroscopic volume $\Omega_A = \Omega_{A-} + \Omega_N$ and thus also depend on nucleon properties such as nucleon compressibility K_N^{-1} which can be deduced from alpha scattering experiments on protons. These estimates[30] give for $K_N^{-1} = 1.4 \pm 0.3$ GeV; almost an order of magnitude greater then the nuclear compressibility $K_A^{-1} = 215$ MeV measured[31] in alpha scattering on nuclear targets. The same experiment[30] found that possible effects of "swelling" or "shrinking" of a nucleon in the nucleus must be very small. These two quantities at equilibrium can be expressed as:

$$K_A^{-1}|_{p_H=0} = 9 \frac{\partial p_H}{\partial \varrho} = -9(A/\varrho^2) \frac{\partial p_H}{\partial \Omega_A} \qquad K_N^{-1}|_{p_H=0} = 9 \frac{\partial p_H}{\partial \varrho_N} = -9(\Omega_N^2/N_q) \frac{\partial p_H}{\partial \Omega_N}$$
(5)

Please note, that the "external" pressure $p_H \doteq -\partial M_N/\partial \Omega_N$ used in (5) is identical with the nuclear pressure $p_H \doteq -\partial M_A/\partial \Omega_A$ appearing in (1,3). The volume changes $\delta\Omega_A, \delta\Omega_N$ for the same increase of pressure are proportional to K_A, K_N respectively. For large $K_N^{-1} \gg K_A^{-1}$ we have (5) $\delta\Omega_A \gg (\Omega_N/\varrho)^2 A \delta\Omega_N > A \delta\Omega_N$ and actually the pressure will mostly squeeze the space between the nucleons, $\Omega_{A-} = \Omega_A - A\Omega_N$. In the limit of constant Ω_N , p_H can be written as:

$$p_H|_{\Omega_N} \doteq -\partial M_A/\partial \Omega_A|_{\Omega_N} = -\partial M_A/\partial \Omega_{A-}|_{\Omega_N} . \tag{6}$$

The total enthalpy $H_A^T \doteq H_{A-} + A(H_N - M_N)$ and using (1,3,6,8) we arrive at the HvH relation (4) for NM containing fixed sized nucleons:

$$H_A^T/A|_{\Omega_N} = \varepsilon - (\partial M_A/\partial \Omega_A)_{A,\Omega_N} (\Omega_{A-}/A + \Omega_N) = \varepsilon + p_H/\varrho = E_F.$$
 (7)

In the next section arguments for a constant nucleon size in compressed NM will be discussed.

1.2. The nucleon mass in the Bag model in NM

In order to consider nucleon volume corrections in the RMF we have to introduce a model for an extended nucleon. Let us take the simplest possibility, the Bag Model. In this model we will discuss whether the nucleon mass M_N as the internal energy or rather a total nucleon energy H_N should be, eventually, constant - independent of the density inside the compressed medium. Such a question is absent in the standard RMF where nucleons are point-like with a constant mass M_N independent of the pressure inside NM. But nucleons themselves are extended. However, in a compressed nucleon, partons (quarks and gluons) have to do work $W_N = p_H \Omega_N$ to keep the space Ω_N for a nucleon "bag". This will involve functional corrections to the nucleon rest energy, dependent on the external pressure with a physical parameter - the nucleon radius R. Others modifications connected with the finite volume of nucleons, such as correlations of their volumes, will be neglected. We introduce a nucleon enthalpy H_N with the nucleon mass M_{pr} possibly modified in the compressed medium

$$H_N(\varrho) \doteq M_{pr}(\varrho) + p_H \Omega_N \quad with \quad H_N(\varrho_0) = M_N,$$
 (8)

as a "useful" expression for the total rest energy of a nucleon "bag".

Describing nucleons as bags, pressure will influence their surfaces [32, 25, 33, 34, 24]. Finite pressure corrections to the mass can not be described clearly by perturbative QCD [35]. Let us discuss the relation (8) in the simple bag model where the nucleon in the lowest state of three quarks is a sphere of volume Ω_N . In vacuum its energy E_{Bag}^0 [36] is a function of the radius R_0 with phenomenological constants - ω_0 , Z_0 [25] and a bag "constant" $B(\varrho)$:

$$E_{Bag}^{0}(R_0) = \frac{3\omega_0 - Z_0}{R_0} + \frac{4\pi}{3}B(\varrho = 0)R_0^3 \propto 1/R_0.$$
 (9)

The condition

$$p_B = -\left(\partial E_{Bag}^0/\partial \Omega_N\right)_{surface} = 0 \tag{10}$$

for the pressure inside a bag in equilibrium, measured on the surface, gives the relation between R_0 and B, used at the end of (9). E_{Bag}^0 fits to the nucleon mass at equilibrium density ϱ_0 where $p_H = p_B = 0$. (E_{Bag}^0 differs from M_N by the c.m. correction [24]).

In a compressed medium, the pressure generated by free quarks inside the bag [36] is balanced at the bag surface not only by the intrinsic confining "pressure" $B(\varrho)$ but also by the nuclear pressure p_H ; generated e.g. by elastic collisions with other hadron [32, 34] bags, also derived in the QMC model in a medium [25]. In a medium, internal parton pressure p_B (10) inside the bag is equal (cf. [25]) on the bag surface to the nuclear pressure

$$p_{H} = p_{B} = \frac{3\omega_{0} - Z_{0}}{4\pi R^{4}} - B(\varrho) \rightarrow 4\pi (B(\varrho) + p_{H})R^{4} = 3\omega_{0} - Z_{0}$$
 (11)

and we get the radius R, depending from a sum $(B+p_H)$:

$$R(\varrho) = \left[\frac{3\omega_0 - Z_0}{4\pi(B(\varrho) + p_H(\varrho))} \right]^{1/4}.$$
 (12)

Thus, the pressure $p_H(\varrho)$ between the hadrons acts on the bag surface similarly to the bag "constant" $B(\varrho)$. A mass M_{pr} can be obtained from (9,11):

$$M_{pr}(\varrho) = \frac{3\omega_0 - Z_0}{R} + \frac{4\pi}{3}B(\varrho)R^3 = \frac{4}{3}\pi R^3[4(B + p_H) - p_H] = E_{Bag}^0 \frac{R_0}{R} - p_H \Omega_N.$$
 (13)

The scaling factor R_0/R comes from the well-known model dependence (9) $(E_{bag}^0 \propto 1/R_0)$ in the spherical bag [36]. This simple radial dependence is now lost in (13) and the pressure dependent correction to the mass of a nucleon given by the product $p_H\Omega_N$, identical with the work W_N in (8), is responsible for this and disappears in the following expression for the nucleon enthalpy

$$H_N(\varrho) = M_{pr} + p_H \Omega_N = (R_0/R) E_{Bag}^0 \propto 1/R(\varrho). \tag{14}$$

The nucleon radius $R(\varrho)$ reflects the scale of confinement of partons. Generally, for increasing $R(\varrho)$, $H_N(\varrho)$ (14) (and M_{pr}) decreases, thus part of the nucleon rest energy is transferred from a confined region Ω_N to the remaining space Ω_{A-} (7). This scenario is rather impossible at the equilibrium density where nucleon compressibility is positive[30]. For decreasing R, H_N increases and the mass $M_{pr} = H_N - p_H \Omega_N$ changes (8) according to the EoS. When NN interactions do not change the rest energy $H_N(\varrho) = M_N$ of partons confined inside nucleons, the radius $R(\varrho)$ is constant (14). It requires work W_N to keep a constant volume Ω_N at the expense of the nucleon mass M_{pr} (13) and is obtained (12) for the constant effective pressure $B_{eff} = B(\varrho) + p_H(\varrho) = B(\varrho_0)$. The $B(\varrho) = B(\varrho_0) - p_H$ gradually decreases with pressure and disappears for $p_H(\varrho) = B(\varrho_0) \simeq 60$ MeVfm⁻³ [36]; in favor of strongly correlated colored quarks in the de-confinement phase when $\varrho \approx (0.5-0.6)$ fm⁻³. For constant M_N the bag constant decreases more slowly and disappears for $p_H \simeq 100$ MeVfm⁻³.

The internal pressure $B(\varrho)$, just as $p_H(\varrho)$ (generated by meson exchanges), originates [37] from interactions of quarks. Therefore, by increasing $p_H(\varrho)$ we can expect a corresponding response in $B(\varrho)$. Actually, when pressure p_H in NM is not taken into account ($p_H = 0$ in (12)) the nucleon radius R, in the QMC model [24], increases in NM. However, in the updated [25] QMC model, which takes into account p_H contributions to the bag radius, a dependence of $R(\varrho)$ on density is a specific property of a particular nuclear model or EoS. In particular, for the ZM model [4], which has the realistic value of $K_A^{-1} \simeq 225$ MeV, the nucleon radius remains almost constant up to density $\varrho = 10\varrho_0$ (the volume corrections (13) to the nucleon mass are absent here). However, for the too stiff EOS of the linear (σ, ω) model, they [25] observe a strong increase of the nucleon radius up to density $\varrho = 2\varrho_0$. Such an increase of the radius would diminish the total rest energy H_N (14) and the nucleon mass (13), making the EOS substantially softeras a consistent feedback. Besides, it has been shown in a Global Color Symmetry Model (GCM) [33], that a decrease of $B(\varrho)$ from the saturation density ϱ_0 up to $3\varrho_0$ by 60 MeVfm⁻³, is accompanied by a similar increase of the pressure p_H . Summarizing, the

sum $B(\varrho)+p_H(\varrho)$ weakly depends on density in GCM or QMC models with a reasonably stiff EOS, thus the bag radius remains approximately constant (12). This justifies the choice of constant total rest energy H_N , unchanged by increasing NN repulsion, just opposite to the case with constant M_N which requires an increase of the rest energy $H_N(\varrho)$ (8,14) with decreasing $R(\varrho)$.

2. Results and Discussion

2.1. The nuclear compressibility - a start of the EOS

Before presenting the results for the EOS, let us consider the nuclear compressibility K_A^{-1} at the starting point at $\varrho = \varrho_0$. A recent self-consistent calculation presented in the thorough review [38] of the nuclear compressibility shows that the density dependence of the surface diffuseness of a vibrating nucleus plays a role in determining the nuclear compressibility. Therefore, we can expect that density dependent properties of nucleons embedded in NM will also modify K_A^{-1} . There are two cases when we can easily derive the nucleon compressibility from (5,13): at constant mass M_N - case **A** (and density ϱ_0) or at constant volume Ω_N - case **B**:

$$K_N^{-1}|_{M_N,R\to R_0} = -3\Omega_N^2 \frac{\partial [M_N(R_0/R-1)/\Omega_N]}{\partial \Omega_N} = M_N \simeq 940 \, \text{MeV} \qquad \text{or} \qquad K_N^{-1}|_{\Omega_N} \to \infty. \eqno(15)$$

We have already mentioned that the experimental estimate[30], using the quark sum rule, gives $K_N^{-1} = (1.4 \pm 0.3)$ GeV, in between our two estimates in Eq.(15); while other theoretical models (refs. in [30]) give a broad range of $K_N^{-1} = (0.6 - 3.0)$ GeV estimates. There is an open question whether the nucleon changes its size and/or mass in medium. Therefore, cases A and B will be discussed and compared.

In case **A** the nucleon mass is constant; according to Eq.13 the nucleon radius will decrease in pressure (density) in order to provide the volume energy; see Fig.1 - right panel. However the volume changes depend on the relative compressibility $K_{rel}^{-1} = K_N^{-1}/K_A^{-1}$. The upper limit of a corrected nuclear compressibility $K_{A\Omega}^{-1}$ - which takes into account excluded volume effects, can be obtained in the limit of $K_N^{-1}/K_A^{-1} \gg 1$ when the nuclear compressibility will be scaled like pressure in (6):

$$K_{A_{\Omega}}^{-1}|_{p_{H}=0} \simeq K_{A}^{-1}|_{p_{H}=0} / (1 - \varrho \Omega_{N}) \approx 1.15 K_{A}^{-1}|_{p_{H}=0}$$
 (16)

However, this is only an upper limit, thus a realistic value of the relative compressibility K_{rel}^{-1} is needed here. Anyway, a small increase of nuclear compressibility is expected in the case of the constant nucleon mass.

In case **B**, the nucleon radius is fixed (the arguments were given at the end of section 1.2), thus a volume correction will diminish (13) the effective nucleon mass similarly to the change made by the scalar part $g_S U_S$ of

$$M_{pr}^*(\varrho) = M_N + g_S U_S - p_H(\varrho)\Omega_N, \qquad \varrho \ge \varrho_0$$
 (17)

the mean field potential; see Fig.1 - left panel. This will involve an additional nonlinear

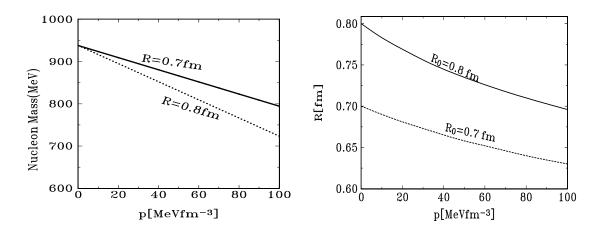


Figure 1. right panel: Case **A** - pressure dependent nucleon radius R for two different initial values of R_0 at the equilibrium density. Left panel: Case **B** - the nucleon mass M_{pr} as a function of NM pressure for two nucleon radii R=0.7fm, 0.8fm.

term $p_H\Omega_N$ in the expression for the energy ε in the RMF ‡. The difference between compressibility $K_{A_{\Omega}}^{-1}$ - with, and K_A^{-1} -without volume correction to energy ε can be written in case **B** as:

$$K_A^{-1} - K_{A\Omega}^{-1} = 9\varrho^2 \frac{\partial^2(p_H \Omega_N)}{\partial \varrho^2} = 9\varrho^2 \frac{\partial^2}{\partial \varrho^2} \left[\frac{r\varrho}{1 - r} \frac{\partial \varepsilon}{\partial \varrho} \right] = \frac{9r\varrho^2}{1 - r} \left[f(\varrho) \frac{d\varepsilon}{d\varrho} + \left(\frac{6 - 5r}{1 - r} \right) \frac{d^2\varepsilon}{d\varrho^2} + \dots \right] (19)$$

where the parameter $r = \varrho \Omega_N$ and $f(\varrho)$ is an unknown regular function at the equilibrium density which multiplies the first derivative $\varepsilon'(\varrho)$ vanishing at ϱ_0 . The last term in (19) with the third derivative is omitted. Consequently, keeping only the term with the second derivative $\varepsilon''(\varrho)$ which is proportional to $K_{A_{\Omega}}^{-1}$, we finally obtain:

$$K_{A_{\Omega}}^{-1}|_{p_{H}=0} \cong \left[\frac{(1-\varrho\Omega_{N})^{2}}{1+4\varrho\Omega_{N}(1-\varrho\Omega_{N})}\right]K_{A}^{-1} \simeq \frac{1}{2}K_{A}^{-1}|_{p_{H}=0}$$
 (20)

that the compressibility $K_{A\Omega}^{-1}$ which includes the volume corrections is smaller by the density dependent factor shown above in (20). The typical saturation density with $\Omega_N \approx 1$ provides a small $\varrho\Omega_N \approx 1/6$ for $R_0 = 0.6$ fm. Consequently $K_{A\Omega}^{-1} \approx K_A^{-1}/2$ (inside uniform NM). It is quite a remarkable reduction which proves how important volume corrections in the vicinity of equilibrium are in the case **B** (constant Ω_N), although energy corrections disappear with pressure for ϱ_0 . Note that the NM compressibility obtained in the the basic Hartree[1] and Hartree Fock [16] calculations give $K_A^{-1} \approx (460 - 560)$ MeV; when it is reduced by a factor 1/2, it arrives at a proper value.

‡ In the realistic Hartree-Fock [16] version of RMF, the volume correction $p_H(\varrho)\Omega_N$ will also modify the following ratio:

$$\frac{M_{pr}^*}{\sqrt{P_N^2 + M_{pr}^{*2}}} = \frac{M_N^* - p_H(\varrho)\Omega_N}{\sqrt{P_N^2 + (M_N^* - p_H(\varrho)\Omega_N)^2}} \simeq \frac{M_N^*}{\sqrt{P_N^2 + M_N^{*2}}}.$$
 (18)

But not far from the saturation density ρ_0 the volume correction can be neglected in this ratio.

2.2. The EoS

The results for energy and pressure of NM in case **A** remain practically unchanged in comparison with the standard classical (σ, ω) RMF calculation with a stiff EOS and large compressibility $K_A^{-1} \simeq 600 MeV$.

At the end of section 1.2 we presented arguments for the second scenario **B** - constant nucleon volume Ω_N (hence constant rest energy $H_N(\varrho) = M_N$), regardless of pressure. In that case we apply the following formulas (8,6) for the nucleon mass M_{pr} modified inside NM with pressure, which is now given by:

$$p_{H} = -(\partial M_{A}/\partial \Omega_{A})_{A,\Omega_{N}} = \varrho^{2} \varepsilon'(\varrho)/(1 - \varrho \Omega_{N}). \tag{21}$$

To carry out calculations we combine the M_{pr} dependence (13,17) of the pressure p_H at the constant nucleon radius $R=R_0$ with the following linear RMF equations [1, 2] for the energy ε in terms of the effective mass M_{pr}^* :

$$\varepsilon = g_V \frac{U_V^0}{2} + \frac{C_2^2}{\varrho} (M_{pr} - M_{pr}^*)^2 + \frac{\gamma}{\varrho} \int_0^{P_F} \frac{d^3 \mathbf{P}_N}{(2\pi)^3} \sqrt{\mathbf{P}_N^2 + M_{pr}^{*2}}$$

$$M_{pr}^* = M_{pr} - \frac{\gamma}{2C_2^2} \int_0^{P_F} \frac{d^3 \mathbf{P}_N}{(2\pi)^3} \frac{M_{pr}^*}{\sqrt{\mathbf{P}_N^2 + M_{pr}^{*2}}}.$$
(22)

 γ denotes the level degeneracy and there are two (coupling) constants: a vector C_v^2 and a scalar C_s^2 , which were fitted at two different saturation points in NM: $S_1[2]$ with $\varrho_0 \approx 0.149 \text{ fm}^{-3}$ and $S_2[1]$ with $\varrho_0 \approx 0.193 \text{ fm}^{-3}$ – see the figure caption. In formula (22) $2C_1^2 = C_v^2/M_N^2$, $2C_2^2 = M_N^2/C_s^2$ with $g_V U_V^0 = 2C_1^2 \varrho$, $g_S U_S = M_{pr} - M_{pr}^*$. Now the finite pressure corrections to M_{pr} (21) convert the recursive equations (22) above the ϱ_0 to a differential-recursive set of equations, taking the general form:

$$f(\varepsilon(\varrho), \varepsilon'(\varrho)) = 0 \text{ for } \varrho \ge \varrho_0.$$
 (23)

Note that (22) is obtained from the energy–momentum tensor for the model Hamiltonian with a constant nucleon mass [1]. Here, we assume that the same equation with mass M_{pr} is satisfied in compressed NM. It should be a good approximation, at least not very far from the saturation density. The corresponding HvH relation (7) for the total nuclear enthalpy: $H_A^T/A = E_F = \varepsilon + p_H/\varrho$, connects the Fermi energy with the nuclear pressure p_H and is met with (0.1-3.)% numerical accuracy; worse for a higher density, ensuring the convergence of our numerical procedure (23).

Linear $(\sigma - \omega)$ models [1, 2] with constant mass M_N produce a too stiff EoS, see Fig.2. Our results, which take into account nucleon volumes, are compared with a semiempirical flow constraint [39] from heavy ion collisions and indeed they correct the EOS making it much softer. Fig.2 shows a good course of the EoS in NM for $\{R_0 = 0.7$ fm, set $S_2\}$ up to a density $\varrho = 0.6$ fm⁻³. In fact, around this density, (partial) deconfinement is expected, which will change the EoS above a phase transition [41]. For $\{R_0 = 0.55$ fm, set $S_2\}$ the EoS is relatively stiffer, slightly above the semi experimental flow constraint. However, it is a good candidate to investigate closely compact stars [42] in the case when hyperons "soften" [12, 43] the EoS further. In Fig.2, it can be seen

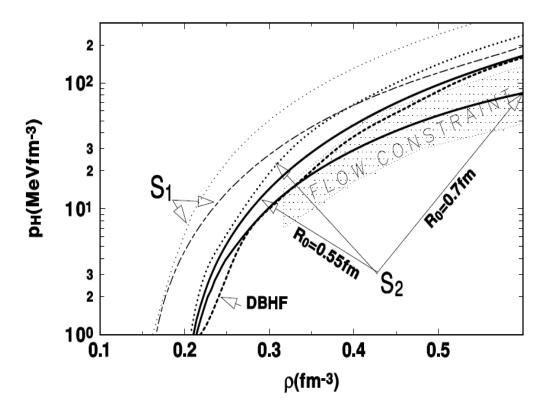


Figure 2. Dotted lines show the pressure for a constant nucleon mass M_N (case A) as a function of the density for two different parameterizations of the (σ, ω) RMF model: $S_1[2]$ with $P_F = 1.30 \text{ fm}^{-1}$ and $S_2[1]$ with $P_F = 1.42 \text{ fm}^{-1}$. The long dashed line shows results for case B for constant nucleon radius $R_0 = 0.7$ fm and S_1 parametrization. Results for set S_2 with $R_0 = 0.55$ fm or $R_0 = 0.7$ fm, marked as solid lines, are indicated by arrows. The phase transition to quarks, for $p_H \gtrsim B(\varrho_0) \approx 60 \text{ MeV}$, is not considered (large B limit). The area indicated by "flow constraint" taken from [39] determines the allowed course of the EoS, using an analysis which extracts from the matter flow in heavy ion collisions from the high pressure obtained there. The DBHF [40] calculations with the Bonn A interaction are shown as short dashes.

that both results for set S_2 are rather close to the DBHF results [40] which produce an EoS able to describe [44] the mass of "PSR J16142230" or "PSR J0348+0432" stars[45] (for $R_0 = 0.7$ fm slightly below the DBHF for higher densities). It is worth mentioning that in the DBHF model there is additional correction [40] from the self-energy, which also diminish the nucleon mass with density. Alternatively, for an additional softening of the EOS the S_1 parametrization with our corrections (dashed line) can be considered.

In our model a volume part $p_H\Omega_N$ (21) originating from the constant total rest energy $H_N = M_N$ (8,14), effectively diminishes the nuclear compressibility, changing its value from the unrealistic $K_A^{-1} = 540$ MeV [1] (set S_2) to the reasonable $K_A^{-1} =$ 328; 252; 172 MeV obtained in our model for $R_0 = 0.6$; 0.7; 0.8 fm respectively, which is in good agreement with our estimate (20). Therefore, the nucleon volume Ω_N is an important physical factor which strongly reduces the compressibility (20) and stiffness in case **B**.

3. Conclusions

It has been shown how the nucleon properties may affect the nuclear compressibility and EoS in the RMF model. Similar changes are expected in the Hartree-Fock version [16, 13] because the nonlinear corrections to energy are almost the same (18).

In case **A**, with a constant nucleon mass and a decreasing radius (13), the changes in the EoS are negligible. However, because the nucleon radius decreases, the deconfinement transition is shifted from $p_H \approx 60 MeV fm^{-3}$ (case **B**) to the higher density $\rho > 0.6 fm^{-3}$ - where the bag constant finally disappears (11) at a pressure $p_H \approx 100 MeV fm^{-3}$.

In case **B** the nucleon radius is constant and the nucleon mass (13) $M_{pr}(\varrho)$ occurred to be a pressure functional. Such a weak dependence of R on ϱ is consistent[25] with the phenomenological EOS. In this case we have found that the total rest energy H_N of the nucleon is independent of density (14), therefore the nucleon mass (13) decreases with ϱ and pressure. It effectively corresponds to nonlinear modifications of a scalar potential. Not accidentally, in the widely used standard [15, 12, 13, 14] RMF model with point-like nucleons the good compressibility is fit by nonlinear changes of a scalar mean field, using two additional parameters (also introducing the density dependent meson coupling constants [18]). Our results indicate that the so called "excluded volume corrections" are the real source of nonlinear modifications in the nuclear medium with finite pressure, which enables calculations with a smaller number of free parameters. The compressibility [46] is lowered to an acceptable value, giving a good course of the EoS for higher densities.

So far, the experimental compressibility K_N^{-1} indicates that we are in between these two scenarios. The presented model is suitable for studying heavy ion collisions and neutron star properties (mass-radius constraint); especially the most massive known neutron stars[45] recently discovered and we plan to include the Fock term and the octet of baryons in future calculations.

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